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# Relativistic light-electron interactions

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**Relativistic Regime** 

**One particle in a Laserfield** 

Many particles in Laserfield => Laser Plasma Interaction

**Particle acceleration:** 

**Wakefield Acceleration** 

**Bubble acceleration Regime** 



## **Regimes of Nonlinear Optics**





# Particle in a Laserfield

**Some definitions:** 
$$\vec{E} = -\partial_t \vec{A} - \nabla \varphi$$
  $a = \frac{eA}{m_0 c} = -\frac{eE}{m_0 c \omega}$ 

#### **Intensity of a Laser field:**

Intensity can be defined in confusingly (and surprisingly) different ways.

It is defined as (in general!):  $\vec{I} = \vec{S} = \vec{E} \times \vec{H}$ 

In an em-wave: 
$$\vec{H} = \frac{\vec{E}}{Z_0} \Longrightarrow I = \frac{E^2}{Z_0}$$
 Z<sub>0</sub> is the vacuum Resistivity

Clear so far, here *I* is instantaneous Intensity, but often is *I* defined as the time average over one laser period:

$$\bar{I} = \langle I(t) \rangle = \frac{1}{Z_0 T} \int_0^T E^2 dt = \frac{E_0^2}{Z_0 T} \int_0^T \cos^2(\omega t) dt = \frac{1}{2} \frac{E_0^2}{Z_0}$$
$$= \frac{E_0^2}{Z_0 T} \int_0^T \cos^2(\omega t) + \sin^2(\omega t) dt = \frac{E_0^2}{Z_0}$$

for linear polarized light

for circular polarized light



When we talk here about intensity we mean the **time averaged intensity** 

and the laser is always polarized in x-direction and propagates in z-direction:

**SO:**  $\vec{a} = a_0 e^{i(\omega t - kz)} \vec{e}_r$  $I = \frac{1}{2} \frac{E_0^2}{Z_0} = \left(\frac{m_0 c}{e}\right)^2 \frac{1}{2Z_0} \cdot \omega^2 a_0^2 = 2.2 \cdot 10^{22} W/m^2 \cdot a_0^2 \qquad \text{for the common laser wavelength}$ 

of 800nm

But it is also common to give the intensity in  $W/cm^2$  so:

 $I = 2.2 \cdot 10^{18} W / cm^2 \cdot a_0^2$ 



#### **Electron in a Laser Field:**

here we look at the trajectory of a single electron in a laser field and we neglect the back reactions of the electron motion to the laser propagation

$$\vec{F} = q\vec{E}$$
  $m_0 \frac{d\vec{v}}{dt} = q\vec{E} \implies \vec{v} = \frac{-e}{m_0} \int \vec{E}dt = \frac{e\vec{A}}{m_0} = c\vec{a}$ 

a>1 => v>c something is wrong!!

a>1 => relativistic regime => we need to include the magnetic Field

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \vec{p}(t) = m(t)\vec{v}(t) = m_0\gamma(t)\vec{v}(t) \qquad \gamma(t) = \frac{1}{\sqrt{1 - \frac{v^2(t)}{c^2}}}$$



A plane wave, polarized in x-direction and propagates in z-direction:  $\vec{a} = a_0 e^{i(\omega t - kz)} \vec{e}_x$ 

$$\frac{dp_x}{dt} = -e\left(E_x - v_z B_y\right) \qquad B = \mu_0 H = \mu_0 \frac{E}{Z_0} = \mu_0 \frac{E}{\sqrt{\frac{\mu_0}{\varepsilon_0}}} = \frac{E}{\sqrt{\frac{1}{\varepsilon_0 \mu_0}}} = \frac{E}{c}$$
$$\frac{dp_z}{dt} = -ev_x B_y$$

Transforming into a moving frame

$$\begin{split} \xi &= z & \partial_z &= \partial_{\xi} - 1/c \,\partial_{\tau} \\ \tau &= t - z/c & \partial_t &= \partial_{\tau} \end{split}$$

we obtain for 
$$v_x$$
:  
 $\partial_{\tau} v_x = -\frac{e}{m_0} E + \frac{ev_z}{m_0 c} E$   
 $v_x = -\frac{e}{m_0} \int E d\tau + \frac{e}{m_0 c} \int E v_z d\tau$   
 $v_x = \frac{eA}{m_0} - \frac{eA}{m_0 c} v_z = ca - v_z a$ 



## Particle in a Laserfield

and for  $v_z$ :

$$\begin{aligned} \partial_{\tau} v_{z} &= -\frac{e}{m_{0}} v_{x} B_{y} = -\frac{e}{m_{0} c} v_{x} E\\ \partial_{\tau} v_{z} &= -\frac{e}{m_{0} c} \left( \frac{e}{m_{0}} A E - \frac{e}{m_{0} c} v_{z} A E \right) = -\frac{e^{2}}{m_{0}^{2} c} \left( -\frac{1}{2} \partial_{\tau} A^{2} + \frac{v_{z}}{c} \frac{1}{2} \partial_{\tau} A^{2} \right)\\ v_{z} &= \frac{e^{2} A^{2}}{m_{0}^{2} c} \frac{1}{2} - \frac{e^{2} v_{z} A^{2}}{m_{0}^{2} c^{2}} \frac{1}{2} = \frac{ca^{2}}{2} - \frac{v_{z} a^{2}}{2}\\ v_{z} &= \frac{ca^{2}}{2 + a^{2}} \end{aligned}$$

So finally  $v_x$  becomes:

$$v_x = ca - \frac{ca^2}{2 + a^2}a = \frac{2ca}{2 + a^2}$$



# Particle in a Laserfield

Electron in a Laser Field:

$$v_x = \frac{2ca}{2+a^2} \qquad v_z = \frac{ca^2}{2+a^2}$$

$$\gamma = 1 + \frac{a^2}{2}$$

$$E_{kin} = m_0 c^2 (\gamma - 1) = m_0 c^2 \frac{a^2}{2}$$

$$E_{kin} = m_0 c^2 (\gamma - 1) = m_0 c^2 \frac{a}{2}$$

**a<<1:** 
$$v_x \approx ca$$
  $v_z \approx \frac{ca^2}{2}$ 

**a>>1:**  $v_x \approx 0$   $v_z \approx c$ 



5 fs, 5\*10<sup>18</sup> W/cm<sup>2</sup>





After the interaction of the electron with the plane Laser wave it has its original velocity

#### $\Rightarrow$ no energy gain in a plane em-wave!

Also from energy and momentum conservation when a photon hits an electron:

after absorption: 
$$\Delta E_{el} = E_{ph} = \hbar \omega, \quad \Delta p_{el} = p_{ph} = \frac{\hbar \omega}{c}$$
  
But:  $E_{ph} = cp_{ph} \Rightarrow \frac{dE_{ph}}{dp_{ph}} = c$   
 $E_{el} = \sqrt{m_0^2 c^4 + p_{el}^2 c^2} \Rightarrow \frac{dE_{el}}{dp_{el}} = \frac{p_{el} c^2}{\sqrt{m_0^2 c^4 + p_{el}^2 c^2}} < c$ 



## Particle in a Laser field





# Particle in a Laser field

#### **Ponderomotive Potential & Force :**

For studying a particle in a Laser field it is convinient to average over the fast oscillating laser field if  $\tau_{Laser} >> 2\pi/\omega \approx 2-3fs$ 

$$U_P = \langle E_{kin} \rangle = m_0 c^2 \langle (\gamma - 1) \rangle = m_0 c^2 \left\langle \frac{a^2}{2} \right\rangle = m_0 c^2 \frac{a_0^2}{4}$$
$$U_P = \frac{e^2}{m_0} \frac{E_0^2}{4\omega^2}$$
$$\vec{F}_P = -\vec{\nabla} U_P = -\frac{e^2}{m_0} \frac{1}{4\omega^2} \vec{\nabla} E_0^2$$

Independent of the charge, particles are pushed out of the Laser focus



What happens, if the electron density becomes higher

Electrons interact with the laser and modify the fields:

 $\vec{\nabla}$ 

$$\vec{\nabla} \times \vec{H} = \vec{J} + \partial_t \vec{D}$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \partial_t \vec{E}$$
$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \mu_0 \vec{J} + \frac{1}{c^2} \left( -\partial_t^2 \vec{A} - \vec{\nabla} \partial_t \varphi \right)$$
$$\vec{\nabla} \left( \vec{\nabla} \cdot \vec{A} \right) - \vec{\nabla}^2 \vec{A} = \mu_0 \vec{J} + \frac{1}{c^2} \left( -\partial_t^2 \vec{A} - \vec{\nabla} \partial_t \varphi \right)$$
$$\vec{\nabla}^2 \vec{A} - \frac{1}{c^2} \partial_t^2 \vec{A} = -\mu_0 \vec{J} + \vec{\nabla} \left( \partial_t \frac{\varphi}{c^2} - \vec{\nabla} \cdot \vec{A} \right)$$

The coupling between electrons and fields is *J*: Here *J* comes from free electrons  $\vec{J} = \rho \vec{v} = -env_x = -enc \frac{2a}{2+a^2}$ 



$$\partial_z^2 a - \frac{1}{c^2} \partial_t^2 a = \mu_0 enc \frac{2a}{2 + a^2} \frac{e}{m_0 c}$$
$$\partial_z^2 a - \frac{1}{c^2} \partial_t^2 a = \frac{\omega_p^2}{c^2} \frac{a}{1 + \frac{a^2}{2}}$$

Were we introduced the so called plasmafrequency:

$$\omega_P = \sqrt{\frac{e^2 n(a(t))}{\varepsilon_0 m}}$$

Assume first a <<1 => 
$$\omega_P = \sqrt{\frac{e^2 n}{\varepsilon_0 m}} = const.$$
  $\frac{a}{1 + \frac{a^2}{2}} \approx a$ 

Ansatz:  $\vec{a} = a_0 e^{i(\omega t - kz)} \vec{e}_x = \partial_z^2 a = -k^2 a, \ \partial_t^2 a = -\omega^2 a$ 



$$-k^2a + \frac{\omega^2}{c^2}a = \frac{\omega_p^2}{c^2}a$$

From this we get the dispersions relation for a em-wave propagation in an electron gas:

$$\omega_p^2 = \omega^2 - k^2 c^2$$

$$k = \frac{\omega}{c} \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \qquad \Rightarrow \text{for } \omega > \omega_p \text{ ,underdense"} \\ \Rightarrow \text{for } \omega < \omega_p \text{ ,overdense"; no wave propagation}$$

Phasevelocity:

$$v_{ph} = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} > c!!$$

Don't panic! Energy and "Information" propagate with the Groupvelocity!!

$$v_g = \frac{d\omega}{dk} = c_v \sqrt{1 - \frac{\omega_p^2}{\omega^2}} < c$$



What happens if *a* becomes larger?

$$\partial_z^2 a - \frac{1}{c^2} \partial_t^2 a = \frac{\omega_p^2}{c^2} \frac{a}{1 + \frac{a^2}{2}} \approx \frac{\omega_p^2}{c^2} a \cdot \left(1 - \frac{a^2}{2}\right)$$

 $\Rightarrow$  for high laser fields the current is reduced compared to the linear current



Single electron current

5 fs, a=1; I=2\*10<sup>18</sup> W/cm<sup>2</sup>





5 fs, a=2; I=8\*10<sup>18</sup> W/cm<sup>2</sup>









This current should produce huge hamonic radiation, but ist this truthly the case?

NO! Because: 
$$\partial_z^2 a - \frac{1}{c^2} \partial_t^2 a = \underbrace{\omega(t)_p^2}_{c^2} \frac{a}{1 + \frac{a^2}{2}}$$

Laser pushes electrons towards low E-Fields (Ponderomotive Force) so:

$$\vec{\nabla} \cdot \vec{J} + \partial_t \rho = 0$$
  

$$1D : \partial_z (nv_z) + \partial_t n = 0$$
  

$$\partial_z (nv_z/c) + \partial_\tau n = 0$$
  

$$\partial_\tau (-nv_z/c+n) = 0$$
  

$$n - nv_z/c = n_0$$
  

$$n(t) = \frac{n_0}{1 - v_z/c} = n_0 \left(1 + \frac{a^2}{2}\right)$$



5fs, a=10; I=2\*10<sup>20</sup> W/cm<sup>2</sup>





So far only an electron gas was considered, but normaly a high intensity laser pulse propagates through a plasma with consists of (nearly inmobile) Ions.

How do we have to modify the propagation equation and what happens to the plasma?

$\frac{dp_x}{dt} = -e\left(E_x - v_z B_y\right)$	
$\frac{dp_z}{dt} = -ev_x B_y - \underbrace{eE_z}$	

The longitudinal electric field comes from density variations:

$$\partial_z E_z = -\frac{e}{\varepsilon} (n - n_0)$$

The same calculations as for the single particle:

$$v_{z} = \frac{e^{2}A^{2}}{m_{0}^{2}c}\frac{1}{2} - \frac{e^{2}v_{z}A^{2}}{m_{0}^{2}c^{2}}\frac{1}{2} - \frac{e}{m_{0}}\int E_{z}dt = \frac{ca^{2}}{2} - \frac{v_{z}a^{2}}{2} + c\phi$$

$$\phi = \frac{e}{m_{0}c}\varphi = -\frac{e}{m_{0}c}\int Edt$$

$$v_{z} = \frac{ca^{2} - 2c\phi}{2 + a^{2}}$$



# Laser plasma interaction

$$v_{x} = \frac{2ca(1-\phi)}{2+a^{2}} \qquad v_{z} = \frac{ca^{2}-2c\phi}{2+a^{2}} \qquad n = \frac{n_{0}}{1-v_{z}/c} \qquad \phi = \omega_{P}^{2} \int dt' \int dt'' (n/n_{0}-1)$$

When *a* is small =>  $v_z$  is small =>  $n \sim n_0$  =>  $\phi \sim 0$  $\Rightarrow$ NO difference in the propagation equation!!!

=> a<<1:

$$\partial_z^2 a - \frac{1}{c^2} \partial_t^2 a = \frac{\omega_p^2}{c^2} a \qquad \omega_p^2 = \omega^2 - k^2 c^2$$
$$v_{ph} = \frac{c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} \qquad v_g = c \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$



For a>>1 the laser significantly pushes the electrons, which form together with the ions a longithudinal E-Field. After releasing by the laser, the electrons oscillate around their origin:  $5fs, a=1; I=2*10^{18} \text{ W/cm}^2$ 





# Laser plasma interaction







1D Simulation of Laser-Plasma Interaction:

5fs, 5\*10<sup>18</sup> W/cm<sup>2</sup>







Tajima, Dawson, PRL43, 267 (1979)



Injected electrons can be accelerated by this wakefield:

$$v_{g,l} = c_{\sqrt{1 - \omega_p^2 / \omega_0^2}}$$
  
$$\Rightarrow \frac{\omega_0}{\omega_p} = \frac{1}{\sqrt{1 - (v_g / c)^2}} = \gamma_p$$

$$\begin{aligned} v_{ph} &= v_{g,l} = c \sqrt{1 - \omega_p^2 / \omega_0^2}, v_{el} \approx c \\ t_{acc} &= \frac{\lambda_p / 2}{v_{el} - v_{ph}} = \frac{\lambda_p / 2}{c \left(1 - \sqrt{1 - \omega_p^2 / \omega_0^2}\right)} \approx \frac{\lambda_p}{c \left(\omega_p^2 / \omega_0^2\right)} \\ L_{acc} &= c t_{acc} = \frac{\lambda_p}{\left(\omega_p^2 / \omega_0^2\right)} \end{aligned}$$



Dephasing length:

$$L_{acc} = \frac{\lambda_p^3}{\lambda_0^2}$$

Maximum particle energy:

Laser Excitation:

 $E_{\rm max}/E_0 = \frac{a_0^2}{2\sqrt{1+a_0^2/2}}$ 

 $W_{\rm max} \approx \frac{1}{2} L_{acc} e E_{\rm max}$ 

$$E_0 = \frac{m_0 c \,\omega_p}{e}$$

Wavebreaking limit:

$$E_{wb}/E_0 = \sqrt{2\left(\frac{\omega_0}{\omega_p} - 1\right)}$$

$$a_{wb} \approx \sqrt{\frac{2\omega_0}{\omega_p} - 1} \quad for \, \omega_0 >> \omega_p$$



# Summary of the first part:

**1D Laser Plasma model\*:** 
$$\partial_{z}^{2}a_{x} - \frac{1}{c^{2}}\partial_{t}^{2}a_{x} = \mu_{0}env_{x}$$
  $v_{x} = \frac{2ca(1-\phi)}{2+a^{2}}$   $v_{z} = \frac{ca^{2}-2c\phi}{2+a^{2}}$   
 $n = \frac{n_{0}}{1-v_{z}/c}$   $\phi = \omega_{P}^{2}\int dt'\int dt'' (n/n_{0}-1)$ 

This leads to

Excitation of plasmawaves:

$$E_{\rm max}/E_0 = \frac{a_0^2}{2\sqrt{1+a_0^2/2}}$$

for electron acceleration to

$$W_{\rm max} \approx \frac{1}{2} \frac{\lambda_p^3}{\lambda_0^2} e E_{\rm max}$$

1D Propagation effects:  $L_{laser} < \lambda_p$ : Selfphase modulation

Wakefield acceleration calls for low densities = high  $E_{wb}$ , high energy gain

\* see also P.Spangle et. Al PRA 41, 4463 (1990) and/or Lecture Notes by J. Meyer-ter-Vehn



# Summary of the first part:

#### In theory Laser Wakefield acceleration is fine but:

Laser must be shorter than plasma wavelength to efficiently excite a plasma wave

Plasma wave must not break

Long interaction distance => guiding of the laser

Electrons must be injected in phase with the plasma wave

### **So WHY Laser Electron Acceleration?**





#### Advantages of Laser wakefield acceleration:

High acceleration fields: TV/m instead of MV/m

Short acceleration distances: mm instead of km

(Train of) short electron bunches

Low emittance

#### Second Part:

•Relativistic nonlinear optics; what changes in 3D?

•Bubble acceleration regime



# Second part

Particle Accelerator are HUGE. Here München vs. CERN:





# **Relativistic nonlinear optics**



$$n_R = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \sqrt{1 - \frac{\omega_{p0}^2}{\omega^2} \frac{1}{\gamma}}$$

When the intensity is high  $n_R$  becomes high

Self-focussing:  $v_{ph} = c/n_R$ 









4x10<sup>18</sup>W/cm<sup>2</sup>, 200fs



Gahn et al. PRL 83, 4772 (1999)



#### 30 fs, 1J laser pulse

## Electron Acceleration by a Wake Field Forced by an Intense Ultrashort Laser Pulse

V. Malka,<sup>1\*</sup> S. Fritzler,<sup>1</sup> E. Lefebvre,<sup>2</sup> M.-M. Aleonard,<sup>3</sup> F. Burgy,<sup>1</sup> J.-P. Chambaret,<sup>1</sup> J.-F. Chemin,<sup>3</sup> K. Krushelnick,<sup>4</sup> G. Malka,<sup>3</sup>
S. P. D. Mangles,<sup>4</sup> Z. Najmudin,<sup>4</sup> M. Pittman,<sup>1</sup> J.-P. Rousseau,<sup>1</sup> J.-N. Scheurer,<sup>3</sup> B. Walton,<sup>4</sup> A. E. Dangor<sup>4</sup>







Laser Plasma Interaction is a highly nonlinear problem especially for strong and short pulses. Understanding relies on numerical simulation:

#### **Numerical Tool**

What do we want to simulate?

High-Power Laser Pulse ionizes any material and forms a plasma

 $\Rightarrow$  Classical, relativistic and highly nonlinear interaction of charged particles with EM-Radiation

 $\Rightarrow$  particles move on a grid of EM-fields



#### **Numerical Tool here:**

3D-PIC Code for Ideal Laser Matter Interaction

#### ILLUMINATION





 $I_0=5x10^{18}$ W/cm<sup>2</sup>,  $\lambda_p=8\mu$ m,  $w_0=5\mu$ m,  $z=100\mu$ m





# The Bubble Regime

12J, 33fs



A. Pukhov and J. Meyer-ter-Vehn Appl. Phys. B (March 2002)



# The Bubble Regime

Here the effect relies on BREAKING the Plasma wave!

E.g. high density is perfered, hence short pulses are needed

THIS is the desired regime:

- •Huge number of electrons in a highly localized pulse
- •"monoenergetic"
- •High conversion efficiency
- •No injection, hence no synchronization

BUT:

Ultrashort (sub-10fs), high intensity (>10<sup>19</sup>W/cm<sup>2</sup>) pulses are needed

Precise control over a number of parameters



# The Bubble Regime

#### **Applications:**

Electron diffraction

x-rays: FEL

Attosecond pump probe experiments

#### Aims:

Tuneable electron soure with respect to energy, energyspread, bunchlength and emittance



# What you will see next

Simulations to reveal the physics of the bubble regime

- •Why are ultrashort laser pulses needed?
- •Scaling of "bubble" properties

Comparing experiments with simulations

First: vary only the intensity. So fixed is: 5 fs,  $w_0 = 5 \mu m$ ;  $\lambda_P = 8 \mu m$ 

$$\frac{a = 3; \ 2 \cdot 10^{19} \ W/cm^2}{a = 5; \ 5.5 \cdot 10^{19} \ W/cm^2} \qquad a < a_{wb} = 4.36$$

$$\frac{a = 5; \ 5.5 \cdot 10^{19} \ W/cm^2}{a = 10; \ 2.2 \cdot 10^{20} \ W/cm^2} \qquad a > a_{wb}$$



















































# 5fs, a=30, 777TW







## 5fs Summary





5fs Summary

Condition to form a stable bubble

$$a > a_{wb}$$
  $L_L < \lambda_p$   $\lambda_p < 2R_b < 6\lambda_p$   $R_b = w_0 \cdot a^{0.25}$ 

 $R_b \sim E_{acc} \sim a^{0.25} =$  > acceleration is limited => multistages

Short lifetime of the bubble ( $\sim$ 200-400µm)

Density profile changes electron bunch properties

Conditions out of reach for present laser systems => but



# Experiments



CCLRC Data, Kyoto (2004)



# Experiments



500mJ, 2.5x10<sup>18</sup>W/cm<sup>2</sup>, 40 fs,  $\lambda_P = 8\mu m$ ,  $\Rightarrow L_L \approx \lambda_p$ 

Mangles et.al, Nature 431, 535 (2004)



# Experiments

$$30 fs, 1J, 3.2 \cdot 10^{18} W/cm^2, \lambda_P = 13.6 \mu m$$



Faure et.al, Nature 431, 541 (2004)



# Simulate Experiments

#### Setup of the Rutherford ASTRA: 1.3\*10<sup>18</sup>W/cm<sup>2</sup>, 40fs

Setup of the Jena Experiment: 2\*10<sup>19</sup>W/cm<sup>2</sup>, 80fs & 40fs





# 40fs, a=0.767, 390mJ











## $a = 0.767; 1.3 \cdot 10^{18} W/cm^2; 390 mJ, \lambda_P = 8 \mu m \quad L_L \approx \lambda_p \qquad a < a_{wb}$























#### New Acceleration Regime (bubble regime)

Unique beam properties: ~10fs, ~1nC, 10<sup>9</sup> Electrons, Divergence ~1°

High conversion efficiency: ~20-35%

#### **Benefits from sub-10fs pulses:**

Tuneable electron source (between 10 and 300MeV) by warying Power/Intensity/Focus/Gas density & Length

Stable in operation

Precise control of gas density and length is necessary